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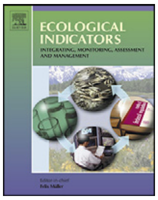

Korcak dimension as a novel indicator of landscape fragmentation and re-forestation

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ABSTRACT

In spatial ecology, habitat fragmentation is an important problem since its increase may create habitat remnants threatening species survival. There are several descriptors to characterize the processes leading to fragmentation. Some of them are model-dependent, while others suffer from the combined error of the perimeter and area measurements of the fragmented patches. In this article – using a theoretical model and a worked example – we would like to show that the Korcak-plot (and the corresponding fractal dimension, the Korcak-dimension) is not only a proper way to describe patchiness, but also applicable to detect secondary processes, like re-forestation, following the primary fragmentation.

1. Introduction

In ecological remote sensing, studying sets of black-and-white patches embedded into 2-D is a frequent problem. Such sets (i.e. group of objects) can be of spatial origin (e.g. an image composed of different patches detected at the same time) or temporal origin (e.g. a series of images from the same patch in a time-span). Using fixed resolution, the most frequent measures of these sets are obtained by measuring the various size-distributions, typically the perimeter and area of the patches. From these distributions one can describe the set with several indices including the various fractal dimensions and compactness.

Digital geometry deals with the geometric properties of digital pictures as well as the approximate geometric properties of digital models representing 2-D and 3-D Euclidean objects (Klette and Rosenfeld, 2004). In previous articles, it has been shown that during the transformation/projection of a real world object into digitalized raster objects, some of the invariance (like translational and rotational ones), which is still maintained for the objects in Euclidean geometry, will be violated (Imre, 2006, 2007). The invariance-violation will be manifested as an un-avoidable error by measuring lengths (perimeters) and areas. The errors in length-based and area-based descriptors are independent, therefore descriptors derived as the combination of perimeters and areas like perimeter–area or Minkowski-dimensions (Mandelbrot, 1982; Imre and Bogaert, 2006) might have quite high errors (containing both of the original errors), while descriptors using only perimeters or areas of the patches will give us results with smaller errors (containing only one of them).

This is the reason why purely area- *OR* purely perimeter-based descriptors might perform much better to describe the behavior of a set of patches embedded into 2-D than other perimeter- *AND* area-dependent descriptors. Additionally, area-based descriptors are usually better that perimeter-based or more generally length-based ones, partly because of their robustness (for example with respect to noise) and partly because of their simplicity: while the area of a digitized patch can be easily calculated by counting its pixels, a perimeter measurement would require a sometimes difficult estimation of the boundary (Bogaert et al., 1999; Ken et al., 2008; Zunic and Martinez-Ortiz, 2009). One of the area-based descriptor is an area–number relation, the so-called Korcak-dimension, firstly introduced by Mandelbrot (1982) and based on the work of Korčák (1938). This is one of the power-law (or fractal) distribution used more and more frequently in ecology (see for example DiBari, 2004 or Thielen et al., 2010). Although the Korcak-law is an empirical one, the Korcak-dimension (see later) seems to be a promising descriptor of fragmented 2D objects and it has been used a few times to describe the size distribution of sets formed by the same physical, chemical, geological process (Hastings and Sugihara, 1993; Sasaki et al., 2006). Compared to other fractal dimensions, the properties of the Korcak-dimension have not yet been well studied but it should be greater than or equal to the Hausdorff-dimension of the same set (Russ, 1994).

Habitat fragmentation is an important problem in ecology since its increase may create habitat remnants threatening species diversity (O'Neill et al., 1988; Turner et al., 1989; Nagendra et al., 2006)

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even if some concerns exist about proofs of evidence of species extinction explicitly due to this ecological process (e.g. Adriaens et al., 2006). Several descriptors exist to estimate this process; some of them are model-dependent, while others might suffer from the previously mentioned errors. Additionally, the footprint of further human activity often can be seen on already fragmented habitats, but sometimes it is controversial to decide what is the effect of a specific activity on a spatial descriptor (Bogaert et al., 2005). The aim of this article is to test the applicability of the Korcak-dimension on remotely sensed imagery to estimate the fragmentation of habitats allowing the individualization of human induced processes accounting for the landscape patterns.

2. Theory behind the Korcak dimension

For our analysis, the so-called Korcak-plot and Korcak-dimension should be introduced first. They were introduced by Mandelbrot (1982), inspired by the work of a Czech scientist, Korčák (1938) in the following form:

$$N(A > a) = ka^{-K} \tag{1}$$

where K is the so-called Korcak-exponent of patchiness, N is the number of patches with area (A) greater than the threshold area (a) and k is a form-factor. For example, having three patches with 8, 9 and 10 units area, $N(A > 7 \text{ units}) = 3$ (i.e. we have 3 patches with area bigger than 7 units), while $N(A > 8 \text{ units}) = 2$ and $N(A > 9 \text{ units}) = 1$. Having statistically similar patches, Mandelbrot (1982) assumed (Seuront, 2010) that:

$$K = \frac{D_K}{2} \tag{2}$$

where K is the Korcak-exponent, D_K is the Korcak-dimension while 2 is the embedding dimension. One should be aware that the Korcak-dimension and the Hausdorff-dimension (i.e. the "classical" fractal dimension) are not identical. In general, the Korcak-dimension is greater than or equal to the corresponding Hausdorff-dimension (Russ, 1994).

In our model, we assume to have a set of patches embedded into two-dimensions with a fragmentation/reforestation process acting in the area. Some patches can be denoted as "forest", others were denoted as "open areas" (arable lands, meadows, etc.). The forests and open area filled the 2-D area only partially; there were other objects (houses, roads, shrublands, other kinds of forest, etc.) separating them and filling the rest of the area.

The effect of re-forestation can be detected in two different ways: through the change of the forest patches or by the change of the open areas (over which ligneous vegetation reforests the area either by natural way or by planned human activity). Otherwise, human induced reforestation (like plantation) generally turns big open areas into new forests. Considering human induced reforestation, imagine that an investor plans to buy open areas in order to turn them with minimized efforts into forest. Assuming limited financial resources, the best solution to obtain ideal compact forest patches would be to buy a few big open patches, rather than a lot of smaller ones. The effect of this way of reforestation would be twofold: (i) an increase of the number of big forest patches and (ii) a decrease of the number of big open areas. Using simple mathematics, we show that Korcak-plots (i.e. the Korcak-dimension) might be able to detect reforestation (or similar secondary processes) on a patchy area, determined by a primary fragmentation process.

3. Theoretical example

Table 1
Threshold area and the number of patches used in the Korcak-analysis. N_o: number of patches above the threshold area in the original set, N_a: number of patches above the threshold area after adding the extra patches, N_r: number of patches above the threshold area after removing the extra patches.

Threshold area	N_o	N_a	N_r
100	100	103	97
137	80	83	77
276	50	53	47
512	33	36	30
1006	21	24	18
2517	12	15	9
9188	5	8	2

Step 1: Let N (the number of patches in Eq. (1)) equals 100, i.e. there are 100 patches embedded into a matrix. The matrix is not homogeneous, but the structure is not relevant for us. To avoid the problems with finite resolution and statistical similarity, we assume that these patches are regular squares, similar rectangles or circles with smooth perimeter (i.e. their perimeter based Hausdorff fractal dimension is 1, while the area fractal dimension is 2). The five largest ones have $9188 + \delta$ unit area (the model is unit independent, it can be in any area unit), where δ is a small number ($\delta \ll 1$), then seven more with $2517 + \delta$ area, 9 with $1006 + \delta$ area, 12 with $512 + \delta$ area, 17 with $276 + \delta$ area, 30 with $137 + \delta$ area and finally 50 with $100 + \delta$ area. From the Korcak analysis, one would obtain that there are 5 patches with an area bigger than 9188 units, $5 + 7 = 12$ with an area bigger than 2517 units, etc. (see Table 1, column N_o stands for number (original)). In the Korcak plot (black full squares in Fig. 1), one will fit a line (solid line) with -0.659 ± 0.004 slope, which means that $D_K = 1.318 \pm 0.008$.

Step 2: In the next step, denote these patches as existing forest and – to model reforestation – add three new ones with the maximal area (i.e. we have 103 forest patches, scattered in a mixed matrix of open lands, roads, etc.). In that case, there will be 8 forest patches with $9188 + \delta$ units area, but still 7 with $2517 + \delta$ area, 9 with $1006 + \delta$ area, etc. For the cumulative numbers for the Korcak plot (i.e. counting the number of patches with bigger area than a threshold one), one will get 8 patches with an area bigger than 9188 units, 15 bigger than 2517 units, etc. (see Table 1, column N_a, which stands for number (added)). The fitting of the double-logarithmic plot of these data (open squares in Fig. 1) would give the slope as -0.568 ± 0.017, which makes $D_K = 1.136 \pm 0.034$. The difference between the two values is significant, but the Korcak-plot stays linear. Therefore, having no preliminary information about the original D_K, one could not distinguish which was the original and which is the new set. We can conclude that the effect of adding a small number of new patches within the range of the biggest area will not cause qualitative change, suggesting

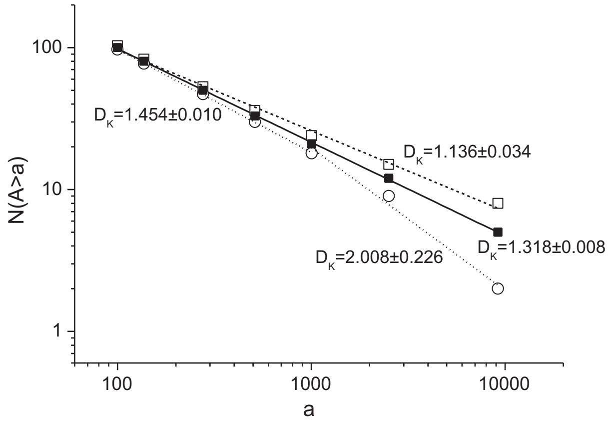

Fig. 1. Korcak-dimension of a set of patches embedded into a 2D matrix (see text). Solid: original set, dashed: set with added new forest patches, dotted: set with removed patches (i.e. with re-forested former open areas).

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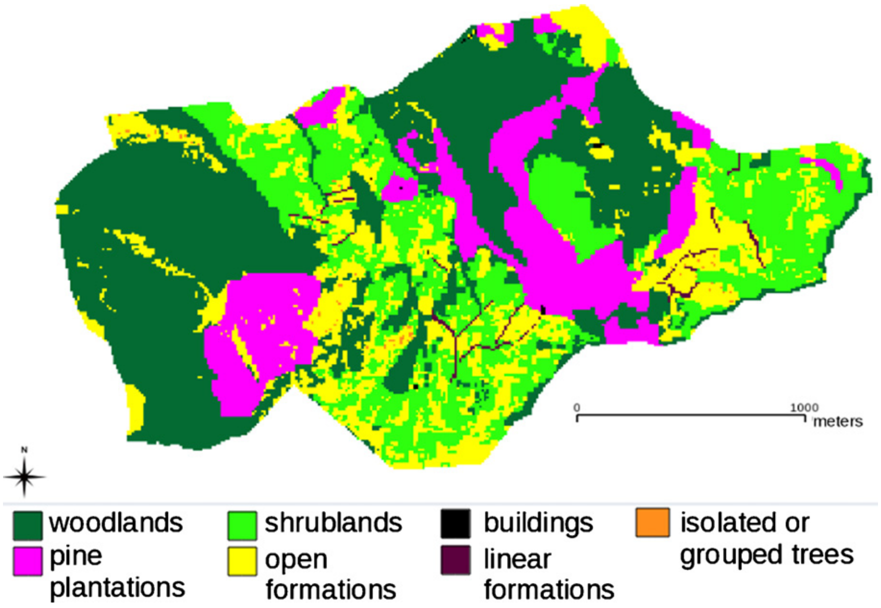

Fig. 2. The land use map of the studied area (Natural Reserve of Poggio all'Olmo, Central Italy) was derived from an aerial photograph taken in 1998. The classification (by a grid of 10 m) was performed after orthorectification of the photograph based on a 10 m digital elevation model and 30 ground control points. Refer to the main text for major information.

that the straightforward way to detect the re-forestation is not good.

Step 3: Now, let us try a new approach. We can denote the patches as "open formations" and the matrix will be a patchwork of different forests, shrublands, roads, etc. In this case, reforestation means the removal of three open area patches from the highest area group (i.e. we have 97 open area patches, scattered in a mixed matrix of different forests, shrublands, roads, houses, etc.). In that case, there will be 2 open area patches with 9188 + δ units area, but still 7 with 2517 + δ area, 9 with 1006 + δ area, etc. For the

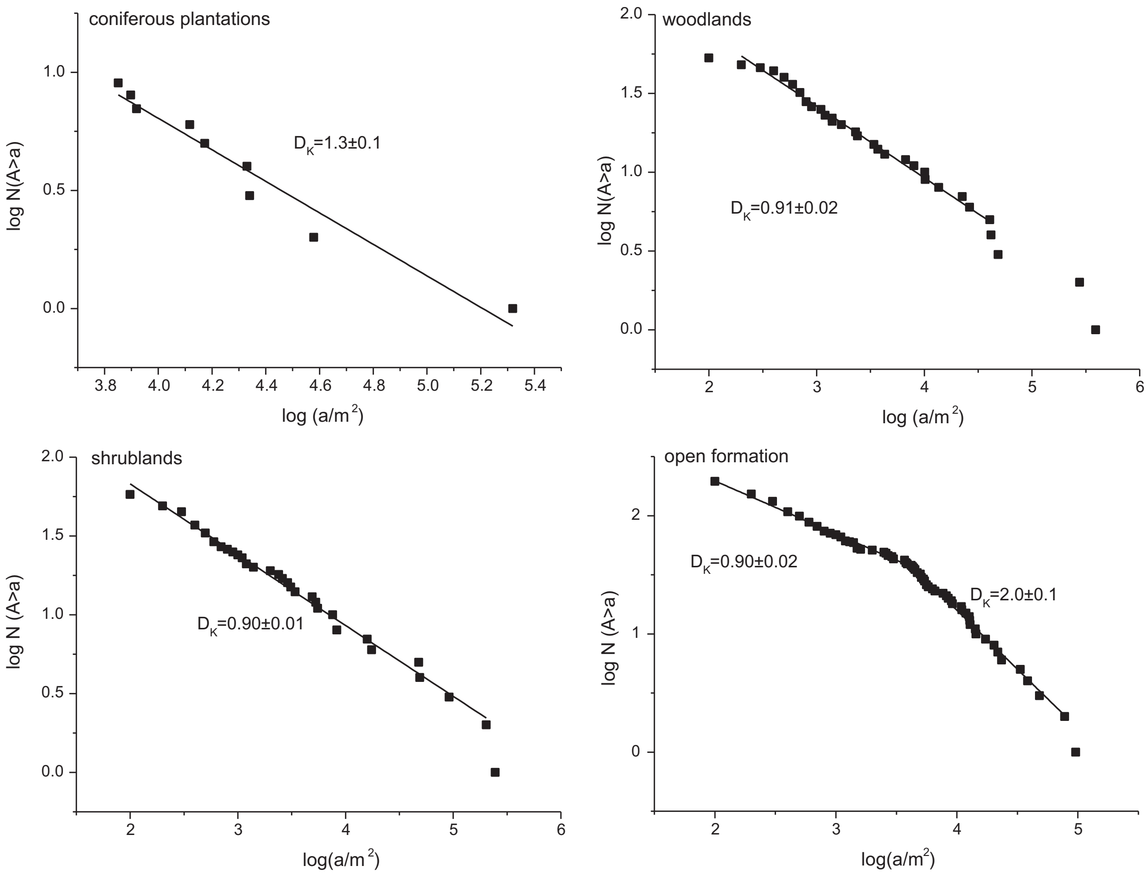

Fig. 3. Korcak-plots of coniferous plantations, woodlands, shrublands and open areas in the 440-ha Natural Reserve of Poggio all'Olmo. Korcak-dimensions are marked on the graphs.

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cumulative numbers, one will get 2 patches with area bigger than 9188 units, 9 with bigger than 2517 units, etc. (see Table 1, column N_r, which stands for number (removed)). It can be seen that this dataset (open circles in Fig. 1) cannot be fit with a single linear fit; one should use higher order fit, or should assume a "virtual" break and use two linear fits for small and for large patches. In our model, the later way is used. We did the latter and obtained two slopes (-0.727 ± 0.006 for small patches, -1.004 ± 0.113 for large patches) and eventually two Korcak dimensions ($D_K = 1.454 \pm 0.01$ for small patches and $D_K = 2.008 \pm 0.226$ for large patches). The first value is not very far from the Korcak-dimension of the original set (i.e. removal of the big patches does not have any effect on the distribution of the small ones), while the second value is a totally artificial value, which can go even above two (which is the upper limit for the Korcak-dimension). In this particular case, we got only $D_K = 2$ (within errors), but removing more big patches (for example one more from the biggest ones and two from the 2517-unit groups) the virtual Korcak-dimension would go well above 2.

The conclusion of this calculation is quite simple. Due to the nature of the logarithmic function, the Korcak-plot is more sensitive to the removal of a few patches than to the addition of the same amount. Therefore the effect of reforestation of some open area might not be seen by plotting the forest patches, but might be seen as a virtual break by plotting the remaining open areas, giving a sensitive tool for us to detect some secondary effect in the fragmentation process.

4. Worked example

This study was initiated by the analysis of different patches (woodlands, coniferous plantations, shrublands and open areas) in the 440-ha Natural Reserve of Poggio all'Olmo on the slope (between 664 and 1016 m) of Mt. Amiata (longitude 11°32′26″E, latitude 42°58′36″N, datum WGS84), Italy (Fig. 2). This is a quite extensively studied area (e.g. Rocchini, 2005; Rocchini et al., 2006; Imre and Rocchini, 2009).

This area underwent a dynamic process of reconstitution of woody vegetation against open formations, e.g. pastures and abandoned fields. Land use changed considerably in the last fifty years in the area of the reserve, because of the depopulation of the countryside and cessation of traditional methods of agriculture after the Second World War. This trend has been reported in a number of studies in the Mediterranean area (see e.g. Rocchini et al., 2009; Geri et al., 2010). The general trend is as follows: after land abandonment open formations are generally composed by patches separated by hedges on the one hand and little homogeneous remnant patches within woodlands on the other. Further, shrubland expands by forming a number of patches by diffuse edges, which over time amalgamate by 'closing' the previously open formations. This phenomenon is basically related to the creation of several nuclei shrubs (tiny groups of shrubs), which initially colonize the open areas in a stochastic manner, becoming increasingly close to one another over the time, before being replaced by woodland patches. Once the woody vegetation cover exceeds a critical threshold this process of colonization will 'self-accelerate' (Loehle et al., 1996).

A grey scaled aerial photo taken in 1998 (scale 1:33,000) was acquired and scanned. Orthorectification, based on a Digital Elevation Model (DEM) derived from a 1:10,000 topographic map (pixel size: 10 m) of the study area and on 30 Ground Control Points (GCPs), was performed using ERDAS IMAGINE 8.4. The final spatial resolution was approximately 2 m. Positional accuracy was tested by means of 20 additional GCPs and never exceeded 4 m. Images were projected into the National (Italian) Coordinate System (Gauss Boaga Projection, datum Roma 40, zone 1).

The Minimum Mapping Unit (MMU) was defined a priori by the superimposition of a vector based grid with a cell size of 10 m × 10 m, giving a total of 44,000 cells; this analysis was conducted using ArcView 3.2 GIS (ESRI). Each cell was subjected to photo-interpretation by means of pixel radiance and the physiognomic characteristics of the contained vegetation. Using this approach, we identified the following land cover classes: woodlands, shrublands, open formations (pastures and fields), buildings, isolated or grouped trees, linear formations (hedges), and coniferous plantations. If a cell contained two or more land cover classes, the value of the prevalent class (in terms of area) was assigned. This operation was carried out on the entire grid. Buildings, isolated or grouped trees and linear formations were too small, and therefore neglected for our analysis.

As it can be seen in Fig. 3, for woodlands, shrublands and for the small patch-area part of the open formations, the obtained Korcak-dimensions are identical within the error, $D_K = 0.90 \pm 0.02$, showing that the process responsible for their formation (primary fragmentation) was more or less the same. For the high patch-size part of the woodlands, there are some disturbances for the data; one can see a discontinuity, which might give some reason to take a look to the open formations. For open formations, a virtual break can be clearly seen around 3000–5000 m^2; below that value D_K is still 0.90, but above that it turns to 2.0 ± 0.1. Obviously one cannot exclude that there might be numerous effect which might be responsible for that virtual break, but based on our model and concerning the recent trends (restoration of the natural habitats, see Rocchini et al., 2006), the most plausible explanation for the break is reforestation, either by humans or by the nature.

Based on the Korcak-plot (Fig. 3) of coniferous plantations (which are also a kind of reforestation, but they are not deriving from the restoration of the original habitat, being the pine not native in that area), the Korcak-dimension is 1.3 ± 0.1, which is clearly higher than the value for the native woodlands, small open formations or shrublands, showing that in the case of plantation, the forming process was a different one, not the natural or random fragmentation and re-growth, but exclusively the human hand. This higher value indicates that intentional human activity (with some plan behind it) tends to produce patches with higher Korcak-dimension, than the natural or random processes.

It should be mentioned here, that there are other analyses for this area, using other, well-established shape and size indices, like mean shape index (MSI), area weighted mean shape index (AWMSI) (Rocchini, 2005) and the Hausdorff-dimension (Imre and Rocchini, 2009), but none of them was sensitive enough to feel the effect of reforestation. According to our results, the Korcak-plot itself is a useful tool to detect secondary processes (like reforestation), while the value of the Korcak-dimension is a promising indicator to distinguish between natural and human-made processes.

5. Conclusion

Habitat fragmentation can be characterized by several descriptors, such as various fractal dimensions or various combinations of size-distributions. Most of these descriptors are derived from the measured perimeters and areas of the fragments (patches). Perimeter measurements are usually more erroneous than area ones, therefore descriptors derived only from patch areas are more accurate that the ones derived from perimeters or from perimeters plus areas. In this short article, the applicability of the Korcak-plot and Korcak-dimension was discussed. For testing, patchiness-data obtained from the Natural Reserve of Poggio all'Olmo, Italy, was used. It was shown, that they are applicable to describe patchiness of fragmented habitats and to detect secondary processes, like re-forestation.

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